Investigation of the thermal model for description of hadron multiplicities in heavy ion collisions

D. Oliinychenko in collaboration with K.A. Bugaev and A.S. Sorin

Dubna, JINR

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- Purpose: description of multiplicities/ratios of hadrons produced in heavy-ion collisions
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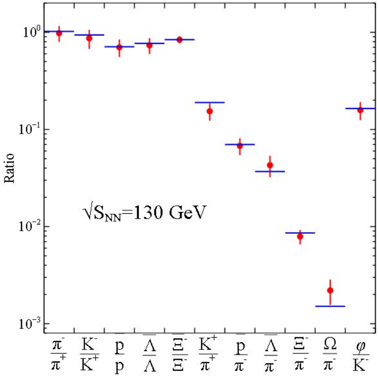
- Grand canonical treatement
- 2 parameters: temperature T and baryo-chemical potential μ_b



Example of our particle ratios description. Points stand for experimental values, lines are theoretical description. $\sqrt{S_{NN}}$ = 130 GeV, T=169 MeV, μ_b = 31 MeV, $\chi^2/NDF = 3.4/9$

Minimal thermal model: brief review

- Purpose: description of multiplicities/ratios of hadrons produced in heavy-ion collisions
- Core assumption: thermal equillibrium
- Grand canonical treatement
- 2 parameters: temperature T and baryo-chemical potential μ_b

Consider hadron gas consisting of h sorts of hadrons at temperature T in volume V. Each sort is characterized by mass m_i and chemical potential μ_i. The number of particles of i-th sort is N_i. Statistics is supposed Boltzmann. Canonical partition function is written as follows:

$$Z_{can}(T, V, N_1, \ldots, N_s) =$$

$$= \prod_{i=1}^{s} \left(\frac{g_i V}{(2\pi)^3} \int exp \left(-\frac{\sqrt{k^2 + m_i^2}}{T} \right) d^3 k \right)^{N_i} \qquad (1.1)$$

Here $g_i = 2S + 1$ is degeneracy factor of i-th hadron sort, k is particle momentum. Grand canonical partition function reads:

$$Z_{gr.can.} = \sum_{N_1=0}^{\infty} \cdots \sum_{N_h=0}^{\infty} exp \left(\frac{\mu_1 N_1 + \cdots + \mu_s N_s}{T} \right) \times$$

$$\times Z_{can}(T, N_1, \ldots, N_s) \qquad (1.2)$$

From (1.2) one gets equillibrium particle quantities of each sort:

$$N_i = \frac{g_i V}{(2\pi)^3} \int exp \left(\frac{-\sqrt{k^2 + m_i^2} + \mu_i}{T} \right) d^3 k \qquad (1.3)$$

Minimal thermal model: brief review

- Conservation laws
- Resonance decays

Conservation of baryon charge B, strangeness S and isospin projection I_3 in average:

$$\begin{array}{c} \mu_i = B_i \cdot \mu_b + S_i \cdot \mu_s + I_{3i} \cdot \mu_{I_3} \\ \sum_{i=1}^{N} n_i S_i = S_{init} = 0 \\ \sum_{i=1}^{N} n_i B_i = B_{init}/V = 200/V \\ \sum_{i=1}^{N} n_i I_{3i} = I_{3init}/V = -20/V \end{array} \qquad (1.4)$$

Here μ_i is the chemical potential of i-th particle sort, n_i is concentration of i-th particle sort, V is total volume.

Minimal thermal model: brief review

- Conservation laws
- Resonance decays
- Hadron width corrections

Resonance decays are accounted for in the following way: final multiplicity consists of the thermal and the decay ones:

$$N_X^{fin} = N_X^{th} + N_X^{decay} = N_X^{th} + \sum_Y N_Y^{th} \cdot BR(Y \to X), \quad (1.5)$$

where $BR(Y \to X)$ is decay branching of Y-th particle into X.

Minimal thermal model: brief review

- Conservation laws
- Resonance decays
- Hadron width corrections
- Excluded volume corrections

Widths of resonances can be taken into account via averaging all expressions, which contain mass, over Breit-Wigner distribution:

$$\int exp\left(\frac{-\sqrt{k^2+m_i^2}}{T}\right) d^3k \rightarrow$$

$$\rightarrow \frac{\int_{M_0}^{\infty} \frac{dx_i}{(x-m_i)^2+\Gamma^2/4} \int exp\left(\frac{-\sqrt{k^2+x^2}}{T}\right) d^3k}{\int_{M_0}^{\infty} \frac{dx_i}{(x-m_i)^2+\Gamma^2/4}}, \quad (1.6)$$

where M_0 is dominant decay channel mass, m is resonanse mass, Γ is resonanse width.

Minimal thermal model: brief review

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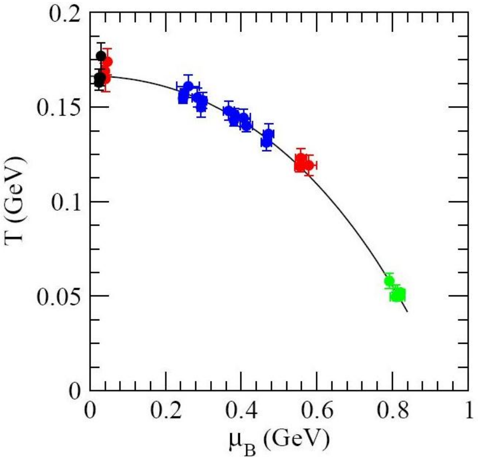
The interaction of hadrons and resonances is usually included by providing a hard core repulsion of Van der Waals-type via an excluded volume correction.

$$p = p_{id.gas} \cdot exp\left(-\frac{pv_0}{T}\right), \quad n_i = \frac{n_i^{id} exp\left(-\frac{pv_0}{T}\right)}{1 + \frac{pv_0}{T}} \quad (1.7)$$

where $v_0 = \frac{2\pi}{3}(2R)^3$ is calculated for a radius considered identical for all hadrons.

Chemical freezeout



Comparison of Chemical Freeze-Out Criteria in Heavy-Ion Collisions; J. Cleymans, H. Oeschler, K. Redlich, S. Wheaton; arXiv:hep-ph/0511094v2 18 Nov 2005;

Section 2

Freezeout criteria discussion

Established freezeout criteria

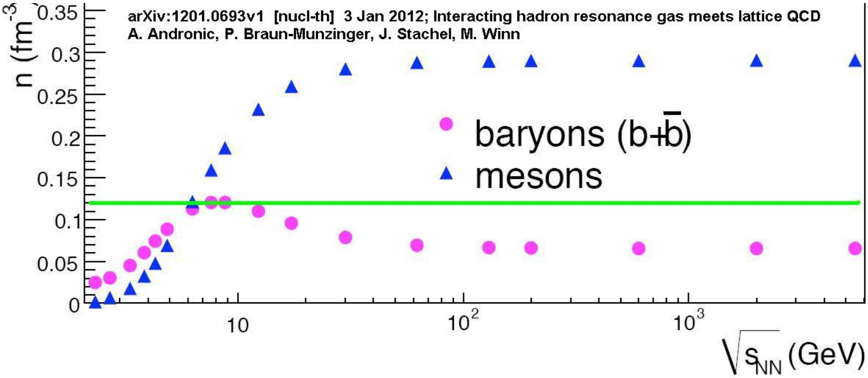
arXiv:hep-ph/0511094v2 18 Nov 2005
Comparison of Chemical Freeze-Out Criteria in Heavy-Ion Collisions J. Cleymans, H. Oeschler, K. Redlich, S. Wheaton

$$n_b + n_{\bar{b}} = 0.12 fm^{-3}$$

$$s/T^3 = 7$$

$$< E > / < N > = 1 GeV$$

Criteria comparison



[1] arXiv:1201.0693v1 [nucl-th] 3 Jan 2012; Interacting hadron resonance gas meets lattice QCD A. Andronic, P. Braun-Munzinger, J. Stachel, M. Winn

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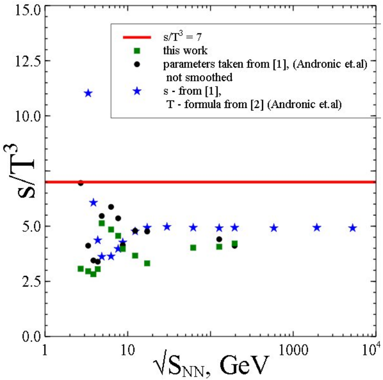

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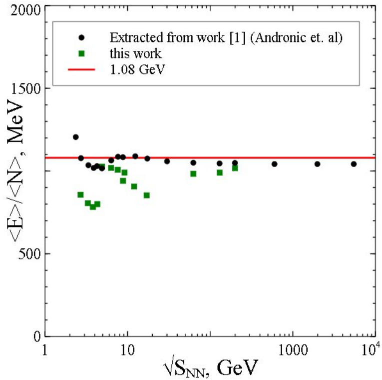

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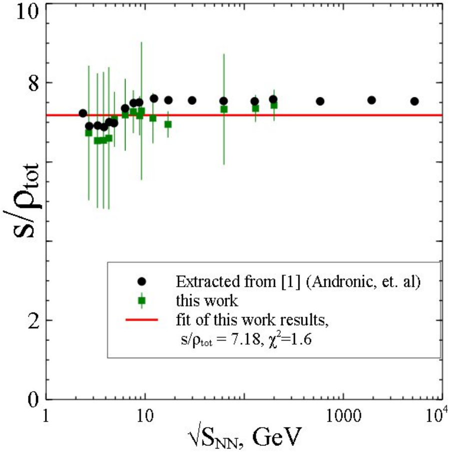
New freezeout criterium

INVESTIGATION OF HADRON MULTIPLICITIES AND HADRON YIELD RATIOS IN HEAVY ION COLLISIONS; D.R. OLIINYCHENKO, K.A. BUGAEV, A.S. SORIN; arXiv:1204.0103v1 [hep-ph] 31 Mar 2012;



Discussion of this freeze-out criterium will be given by K.A. Bugaev in his talk.

Section 3

Conservation laws investigation

Conservation laws: multiplicities fit

- Multiplicities fit: 3 free parameters - T, μ_b and V

$$n_i = n_i(T, \mu_b, \mu_s, \mu_{I_3})$$
$$\sum_{i=1}^{N} n_i S_i = S_{init}/V = 0$$
$$\sum_{i=1}^{N} n_i B_i = B_{init}/V = 200/V$$
$$\sum_{i=1}^{N} n_i I_{3i} = I_{3init}/V = -20/V$$

- System of eq-s: 3 equations, 5 unknowns, 3 of them are fitted $\rightarrow$ one eq-n might be not satisfied
- Barionic sum $S_b = V \cdot \sum_{i=1}^{N} n_i B_i$. If the equation standing for baryon conservation is satisfied, then $S_b = const = 200$.

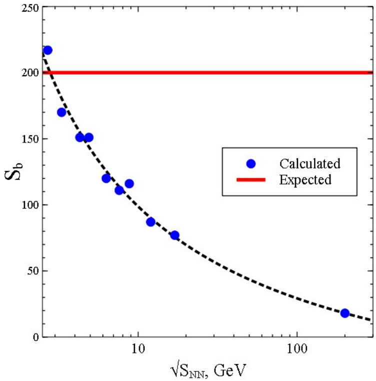


Figure: Baryonic sum $S_b = V \cdot \sum_{i=1}^{N} n_i B_i$ vs. $\sqrt{S_{NN}}$. Blue points are values calculated from baryon conservation, red line is an expected value.

Conservation laws: ratios fit

- Ratios fit: 2 parameters - T, μ_b.
- Volume V is extracted from conservation laws
- Multiplicities: $N = n \cdot V$

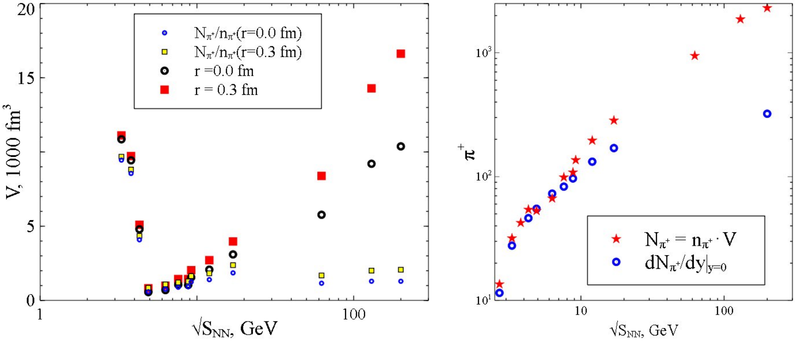


Figure: Left panel: Volume at chemical freezeout, extracted from conservation laws. Two cases of hard-core radii - 0.0 fm and 0.3 fm; Right panel:π^+ multiplicity obtained via multiplying concentration over volume and an experimental multiplicity;

Section 4

Radii fit and Lorentz contraction

Radii global fit

- Model with different meson and baryon radii, R_m and R_b was considered
- Global fit of experimental data was performed
- Restrictions over R_m and R_b were obtained
- Lorentz contraction was also considered

Radii global fit

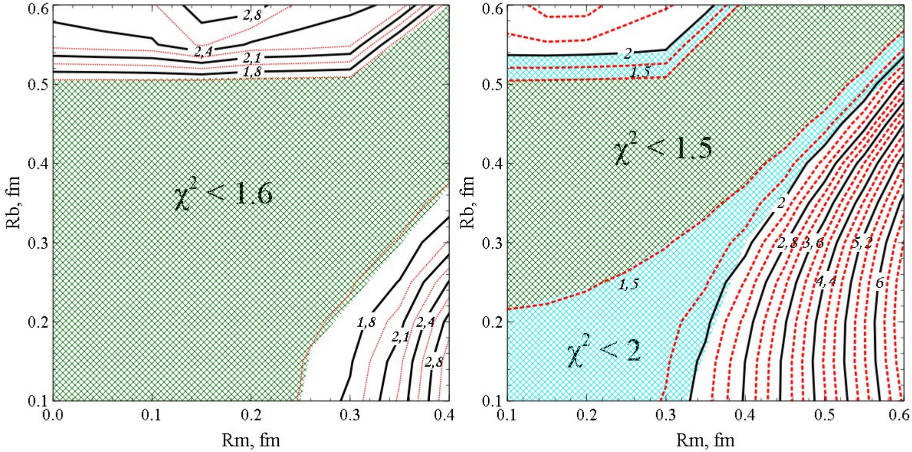


Figure: Left panel: χ^2/NDF, no Lorentz contraction; Right panel: χ^2/NDF with Lorentz contraction;

Lorentz contraction

- Needs no new parameters
- Frees model from causality paradox
- Improves ratios description quality
- Might improve strangeness horn description

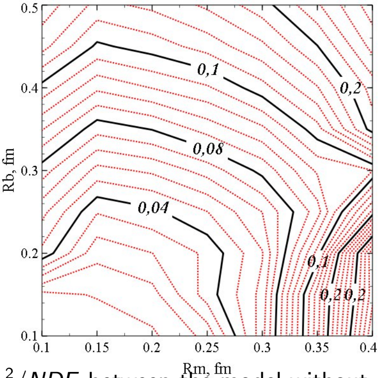


Figure: Difference of χ^2/NDF between the model without contraction and the model with Lorentz contraction.

Lorentz contraction

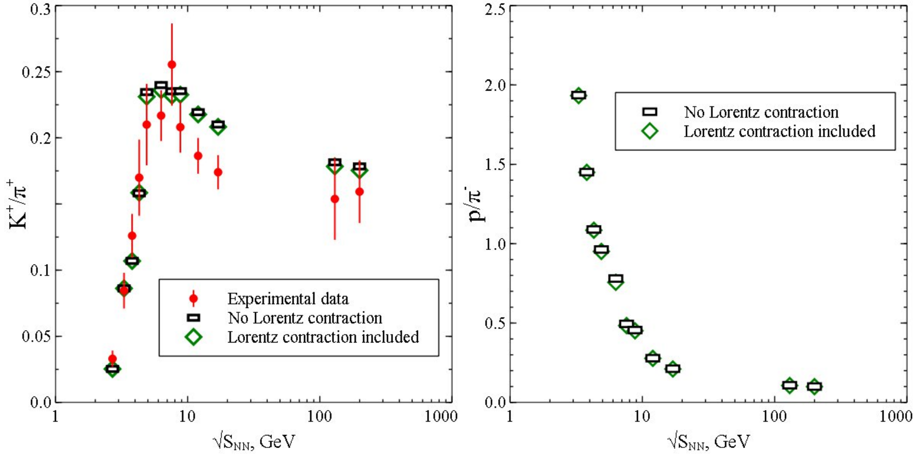


Figure: Left panel: $\frac{K^+}{\pi^+}$; Right panel: $\frac{p}{\pi^-}$ ratio;

Section 5

Summary

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- Conservation laws should be altered
  - For ratios description baryon conservation law can be neglected
  - For multiplicities description 3 fit parameters lead to ambiguity
- Experimental restrictions over baryon and meson radii are obtained

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- New freeze-out criterium $s/\rho_{tot} \approx 7$ was suggested, it is a new physical effect and we call it adiabatic hadron production at freeze-out.

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Thank you!

Thank you for your attention!

Questions are welcome.

Our modification of the thermal model:

Concerns only excluded volume corrections

- Arbitrary number of hadron sorts - s. Arbitrary symmetric excluded volume sxs matrix b, where b_{ij} is excluded volume for sorts i and j.
- Canonical partition function:

$$Z(T,V,N_i) \approx \prod_{i=1}^{s} \frac{\phi_i^{N_i}}{N_i!} \times \left[\prod_{i_1=0}^{N_1-1} (V - b_{11} i_1) \right] \times \left[\prod_{i_2=0}^{N_2-1} (V - b_{12} N_1 - b_{22} i_2) \right] \times \cdots \times \left[\prod_{i_s=0}^{N_s-1} (V - \sum_{j=1}^{s} b_{sj} N_j - b_{ss} i_s) \right] \quad (2.1)$$

where ϕ_i stands for a momentum integral: $\phi_i(T,m,g) = \frac{g}{2\pi^2} \int\limits_0^\infty k^2 exp\left[-\frac{E(k)}{T}\right] dk$, in which $E(k) = \sqrt{k^2+m^2}$ and $g = 2J+1$ is a degeneracy factor, N_i - multiplicity of each sort.

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- We modify CPF (2.1) so that first order over V_{eigen}/V remains the same:

$$Z_{VdW}(T, V, N_i) = \left[\prod_{i=1}^{s} \frac{\phi_i^{N_i}}{N_i!} \right] \times \left[V - \frac{N \cdot b \cdot N^T}{M} \right]^M , \quad (2.2)$$

where $N = (N_1, N_2, \ldots, N_s)$ and $M = \sum_{i=1}^{s} N_i$ is total number of particles.

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Our modification of the thermal model:

Concerns only excluded volume corrections

- Final equation system:

$$\xi_i = A_i exp \left(-\sum_{j=1}^{s} 2\xi_j b_{ij} + \frac{\xi b \xi^T}{\sum_{j=1}^{s} \xi_j} \right), i = 1..s \quad (2.3)$$

Pressure is written as

$$p = \sum_{i=1}^{s} \xi_i \quad (2.4)$$

Here $A_i = \frac{g}{2\pi^2} \int_0^{\infty} k^2 exp \left[-\frac{E(k)+\mu_i}{T} \right] dk$ and

$$\xi_i = \frac{N_i}{V - \frac{N^T B N}{M}} \quad (2.5)$$

,

$$\xi = (\xi_1, \xi_2, \ldots, \xi_s) \quad (2.6)$$

From equations (2.3) one gets concentrations $n_i = \frac{N_i}{V}$:

$$n_i = \frac{\xi_i}{1 + \frac{\xi^T B \xi}{\sum_{j=1}^{s} \xi_j}} \quad (2.7)$$

- Transforms into conventional case for equal $b_{ij} = v_0$.